Robust Auction Design with Support Information

A seller wants to sell an indivisible item to n buyers. The buyer valuations are drawn $i.i.d.$ from a distribution, but the seller does not know this distribution; the seller only knows the support $[a, b]$. To be robust against the lack of knowledge of the environment and buyers' behavior, the seller optimizes over dominant strategy incentive compatible (DSIC) mechanisms, and measures the worst-case performance relative to an oracle with complete knowledge of buyers' valuations. Our analysis encompasses both the regret and the approximation ratio objectives. For these objectives, we derive an optimal mechanism in quasi-closed form, and the associated performance, as a function of the support and the number of buyers n. Our analysis reveals three regimes of support information and a new class of robust mechanisms. i.) With "low" support information, the optimal mechanism is a second-price auction (SPA) with a random reserve, a focal class in the earlier literature. ii.) With "high" support information, we show that second-price auctions are strictly suboptimal, and we establish that an optimal mechanism belongs to a novel class of mechanisms we introduce, which we call pooling auctions (POOL); whenever the highest value is above a threshold, the mechanism still allocates to the highest bidder, but otherwise the mechanism allocates to a uniformly random buyer, i.e., pools low types. iii.) With "moderate" support information, we establish that a randomization between SPA (with a random reserve price) and POOL (with a random threshold) is optimal. We also characterize optimal mechanisms within nested central subclasses of mechanisms: standard mechanisms that only allocate to the maximum value bidder, SPA with random reserve, and SPA with no reserve. We show strict separations in terms of performance across classes, implying that deviating from standard mechanisms is necessary for robustness. Lastly, we show that the same results hold under other distribution classes that capture "positive dependence", namely: $i.i.d.$, mixture of $i.i.d.$, and exchangeable and affiliated distributions, as well as $i.i.d.$ regular distributions.